

ВЛАДИМИР АЛЕКСАНДРОВИЧ БЕЗДЕЛЬНИКОВ
аспирант кафедры прикладной математики и кибернетики
математического факультета, Петрозаводский государствен-
ный университет (Петрозаводск, Российская Федерация)
misterfast@gmail.com

ELASTIC DEFORMATION MODELING BASED ON SURFACE GRID*

A method of modeling three-dimensional object deformation is described. This problem originated from the eye microsurgery operation framework development and thus has some advantages that can help in physical simulation. We suggest an approach that can reduce real-time deformation calculation complexity. The standard way of elastic objects' modeling with a three-dimensional tetrahedron grid does not allow achievement of the goal. Instead of using a three-dimensional grid we simulated the surface reaction by taking into account the internal volume forces' measure.

Key words: Eye surgery simulation, small elastic deformations, elasticity, plasticity, inner pressure approximation

INTRODUCTION

Computer-based medical simulation systems can provide a significant benefit for ophthalmologic microsurgical procedure training. They reproduce a number of complex medical procedures that demand high level of skill and training providing at the same time the required tactile experience. The most difficult part of microsurgery operation modeling refers to dynamic behavior modeling of elastic eye areas as a reaction to external force induced by haptic interface. Previous works in this area have shown good results in the sphere of eye microsurgery modeling [1], [2], [10].

However, when current available solutions propose realistic physical models of the organs, they actually oversimplify them. Most of the existing medical simulators use a flat model of eye areas that allows to recreate an operated object image in a form surgery usually deals with. Thus we are also faced with the problem that sometimes we cannot rely on the simulation of eye areas as thin tissues. Instead, we must consider them as volume objects and simulate tissues cutting. Thus we are developing and using a new platform that realizes new visualization and modeling features [8]. Since microsurgery operation implies rather small displacements of a surgery tool, it is necessary to model fast eye shells reactions without lags. There is a number of works [4], [5], [6], [7] where elastic objects are modeled with a three-dimensional grid. In contrast, we assume that three-dimensional elastic objects can be modeled with a surface grid assuming that internal forces are also taken into account. Some submissions allow to apply this modeling relaxation. Here is the list of the most significant ones:

- **There is no need to make an exact simulation of three-dimensional object dynamics.** In the training system model we use an abstract object, the parameters of which are unknown, but not a real one, which we need to display. Therefore, we can use only general regularities to render an object that plausibly simulates the modeling process.
- **Relatively small consecutive object transformations.** All surgery manipulations represent gradual, relatively slow motions.

ELASTIC DEFORMATION MODELING

Correct deformation model must view an object as a combination of inserted into each other interacting layers. This is usually modeled as a three-dimensional grid [3], [9]. Traditionally an explicit formulation of finite element methods is applied to model deformations of elastic objects. An object is divided into a set of sub-elements tetrahedrons. The general formulation of the finite element method is expressed by an equation:

$$M\ddot{x} + D\dot{x} + Lx = F,$$

where is the displacements of the nodes and M , D , L , F are matrixes of mass, damping, stiffness and external forces, respectively [4]. This means solving equations system for all 3D-grid vertices in real-time [11], which can affect lags and improper modeling quality. Thus we have concentrated on the modeling of object interior behavior as a reaction to external forces by tracing surface S node positions.

Consider first a simple case for which surface S is a flat rectangular grid. This grid can be viewed as a graph $G = \langle V, E \rangle$, the vertexes of which are numbered with indexes (i, j) :

$$V = \{(i, j) \mid -n \leq i, j \leq n\}.$$

Locate first these vertices V in points $A_{ij} = (x_{Aij}, y_{Aij}, z_{Aij}) = (i, j, 0)$ of three-dimensional space. We will assume that boundary grid nodes are fixed in a sense that $A_{ij} = (i, j, 0)$ if $i = \pm n$ and $j = \pm n$ presupposing that the surface is isotropic. Consider two nodes with coordinates (i, j) and (i', j') are adjacent if $|i - i'| + |j - j'| = 1$ and there are elastic links between them with an ability of tension and compression. Other presuppositions describe the relationship between the nodes in the following way. Boundary nodes positions do not depend on external and inner forces. Thus we have boundary condition. Other nodes are inner, i.e. their coordinates can be changed. Assume also that the external forces which are applied to node A_{ij} do not depend on the node position.

Inner nodes (i, j) are linked by elastic forces Fx with horizontally adjacent node $(i \pm 1, j)$. These forces do not depend on the number of nodes, but are

calculated by distance $d = x_{Ai+1,j} - x_{Ai,j}$ between the nodes by the following empirical square principle:

$$F_x(d) = \begin{cases} \gamma_x^+(d-1)^2 + \sigma_x^+(d-1), & \text{if } d-1 > 0 \\ \gamma_x^-(d-1)^2 - \sigma_x^-(d-1), & \text{if } d-1 < 0 \end{cases}$$

In case $d-1 > 0$ horizontal tensile deformations of surface are acting, in other case compression deformations are acting. The same conclusions are valid for node (i,j) that is linked by F_y with vertically adjacent nodes $(i, j \pm 1)$. Here $\sigma_{x,y}^\pm$ and $\gamma_{x,y}^\pm$ are some real numeric parameters. We call these forces F_x and F_y as surface forces; they are always directed tangential to the surface in the current node.

A more complex case is related to the third dimension. Actually a force, which is applied to node A_{ij} , can really affect its displacement by decreasing z-coordinate of the node and possibly the displacement of deeper nodes, according to the laws of physics. The reaction force along z-axis will be compensated by surface forces, but it is obviously not enough. The balloon reaction to touching in some point actually depends on inner pressure rather than on elastic surface properties only. Thus we need to take into account the volume force as a reaction to compression or tension of elastic tissue in the interior of an object. In short, this force is generated by hypothetical inner surface levels that form an object.

Here a problem arises. It is obvious to calculate the surface node reaction in response to compression force that is orthogonal to the surface plane. Besides surface tissue compression, which is insignificant in case of small force, we must take into account the inner tissues reaction. Determine corresponding force $F_z(d)$, where $d = z_{Ai+1,j} - z_{Ai,j}$ is a node (i, j) moving along the z-axis, which is $F_z(d) = \gamma_z^+ d^2 + \sigma_z^+ d$, if $d > 0$, i. e. compression force is applied, and $F_z(d) = \gamma_z^- d^2 - \sigma_z^- d$, if $d < 0$, i. e. tensile force is applied to the node.

We associate the inner volume reaction value with a solid angle between the current and four adjacent nodes. To calculate its value we split it into two trihedral angles. The edges of these angles are calculated by vectors:

$$\begin{aligned} a_1 &= [A_{i,j}, A_{i-1,j}], & a_2 &= [A_{i,j}, A_{i-1,j}], & a_3 &= [A_{i,j}, A_{i-1,j}]; \\ a'_1 &= [A_{i,j}, A_{i-1,j}], & a'_2 &= [A_{i,j}, A_{i-1,j}], & a'_3 &= [A_{i,j}, A_{i-1,j}]. \end{aligned}$$

We determine the value of trihedral angle by formula:

$$t(a_1, a_2, a_3) = \frac{[a_1, a_2, a_3]}{|a_1||a_2||a_3| + |a_1|(a_2, a_3) + |a_2|(a_1, a_3) + |a_3|(a_1, a_2)},$$

where $[a_1, a_2, a_3]$ is a scalar triple product, (a_2, a_3) – scalar product, $|a|$ – norm of vector. We can calculate the value of tensile force by using the two above formulas as $F_z(t(a_1, a_2, a_3) - 2\pi) + F_z(t(a'_1, a'_2, a'_3) - 2\pi)$, where $t - 2\pi$ is a measure of z-axis displacement. Now we can use balance equations to calculate node equilibrium positions.

CONCLUSION AND FUTURE WORK

In this paper we presented the way how to measure elastic object deformation. Here we use the standard computer object representation that is based on triangle. We illustrate on a simple case how to reduce the number of object grid vertices that are taken into account while calculating the surface reaction of the object in response to external force. This approach can be generalized to the case when we have arbitrary convex grid and apply external force to some grid nodes.

The future work will be focused on grid cutting. Here we can encounter difficulties, since we must generate new links and grid nodes for simulating tissues cutting. These new nodes will affect the changes in the above elastic object deformation modeling.

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