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ON A GENERAL RELATIVITY EXTENSION*

В статье «О расширении общей теории относительности» показано, что анализ свойств дуальности тензора кривизны Римана указывает на возможность расширения эйнштейновской общей теории относительности к теории типа неабелевой янг-миллсовской. Уравнения движения новой теории есть уравнения Янга-Миллса для тензора кривизны. Эйнштейновские уравнения (с космологическим членом, появляющимся как константа интегрирования) содержатся в предлагаемой теории. Новое в сравнении с прежней теорией состоит в том, что гравитационные поля не определяются исключительно тензором энергии-импульса материи, а обладают своей собственной неэйнштейновской динамикой (вакуумные флуктуации, самодействие), что вообще типично для неабелевых калибровочных полей.

Ключевые слова: неабелева дуальность, инстантон, неэйнштейновская гравитация, космологический член

INTRODUCTION

There is no doubt that Einstein's General Relativity [2] is a nonabelian gauge theory although it is not quite the same as the conventional Yang-Mills theory [10]. Though this theme is a subject of much controversy since R. Utiyama [9] and T. Kibble [4] proposed the very first gauge models of Gravitation.

Nevertheless, there are rather many arguments in favor of the theory is nonabelian. But how does the fact that gravitation is nonabelian agree with the widely spread and prevailing view the gravity source is energy-momentum and only energy-momentum? And how about nonabelian self-interaction? Of course, here we touch very tender spots about exclusivity of gravity as physical field, the energy problem, etc. Still the spherically-symmetric field out of Schwarzschild's [8] sphere looks quite like Coulomb's solution in Electrodynamics, the abelian theory without self-interaction. All the facts point out the General Relativity is not quite conventional nonabelian theory. In addition, Einstein's equations are not like Yang-Mills'.

It is shown in this paper that the theory can be formulated *ad exemplum* as an ordinary Yang-Mills' theory with more or less standard description in the form of the Yang-Mills equation, with self-interactions and instantons. For all that, Einstein's equations are contained in the theory rather than cancelled and do not dwindle. And their existence as themselves seems to relate to the peculiarities of gravity.

For our purposes, the essential fact is that internal (group) indices and space-time one are interchangeable, i. e. group acts in the Minkowski space-time which as a result becomes curved. In fact the internal space coincides with space-time. Therefore, it is convenient to hold the viewpoint that the first

two indices α, β of the curvature tensor $R_{\alpha\beta\mu\nu}$ are internal, and the second pair μ, ν are the spacetime indices. And vice versa that's right as well. This is the peculiar features of the gravity as a gauge theory. For this reason the gravity duality properties are even more nontrivial and interesting than those in the ordinary Yang-Mills theory.

THE DUALITY PROPERTIES OF THE RIEMANN TENSOR

The duality properties which we are interested in have been established in the article [5] which however includes some mistakes. About notations. The metric with signature $(+, -, -, -)$ in $D = 4$ pseudo-riemannian manifold is given to be metric-compatible to a (Riemannian) connection in the regular way.

Let us introduce the operations: 1) the left dual conjugation ($*R_{\alpha\beta\mu\nu}$), 2) the right dual conjugation ($R^*_{\alpha\beta\mu\nu}$), and 3) twice dual conjugation ($**R^*_{\alpha\beta\mu\nu}$)

$$\begin{aligned} *R_{\alpha\beta\mu\nu} &= \frac{1}{2} E_{\alpha\beta\rho\sigma} R^{\rho\sigma}_{\mu\nu}, R^*_{\alpha\beta\mu\nu} = \\ &= \frac{1}{2} R_{\alpha\beta}^{\rho\sigma} E_{\rho\sigma\mu\nu}, **R^*_{\alpha\beta\mu\nu} = \frac{1}{2} E_{\alpha\beta\rho\sigma} R^{\rho\sigma}_{\gamma\delta} E^{\gamma\delta\mu\nu}, \end{aligned}$$

where $E_{\alpha\beta\mu\nu} = \sqrt{-g} \varepsilon_{\alpha\beta\mu\nu}$ – the Levi-Civita tensor, g – the metric tensor determinant.

For example

$$**R_{\alpha\beta\mu\nu} = R^*_{\alpha\beta\mu\nu} = -R_{\alpha\beta\mu\nu}.$$

It is usual properties of double dual conjugates in the $(+, -, -, -)$ riemannian space.

In terms of the dual conjugates the cyclicity identity

$$R_{\alpha\beta\mu\nu} + R_{\alpha\nu\beta\mu} + R_{\alpha\mu\nu\beta} = 0$$

is of the form

$$*R_{\alpha\mu}^{\nu} \equiv *R_{\alpha\mu}^{\nu} = 0 \text{ and } / \text{ or } *R_{\alpha\mu}^{\nu} \equiv *R_{\alpha\mu}^{\nu} = 0.$$

Bianchi's identity

$$R_{\mu\nu\rho\sigma;\delta} + R_{\mu\nu\delta\rho;\sigma} + R_{\mu\sigma\delta\rho;\nu} = 0$$

transforms to

$$*R_{\mu}^{\nu}{}_{\rho\sigma;\nu} = 0 \text{ and } / \text{ or } *R_{\mu\nu\rho}^{\sigma}{}_{;\sigma} = 0.$$

Or that can be rewritten as follows

$$*R_{\alpha\beta\mu}^{\nu}{}_{;\nu} = 0 \text{ and } / \text{ or } *R_{\alpha}^{\beta}{}_{\mu\nu;\beta} = 0.$$

And twice dual conjugate Riemann's tensor $*R_{\alpha\beta}^{\mu\nu}$ can be represented by Riemann's and its contractions, i. e. by Ricci's tensor and scalar, for the expression

$$E^{\alpha\beta\rho\sigma} E_{\gamma\delta\mu\nu} = \varepsilon^{\alpha\beta\rho\sigma} \varepsilon_{\gamma\delta\mu\nu}$$

can be calculated and expressed by the Kronecker δ -symbols:

$$*R_{\alpha\beta}^{\mu\nu} = -R_{\alpha\beta}^{\mu\nu} + \delta_{\alpha}^{\mu} R_{\beta}^{\nu} + \delta_{\beta}^{\nu} R_{\alpha}^{\mu} - \delta_{\beta}^{\mu} R_{\alpha}^{\nu} - \delta_{\alpha}^{\nu} R_{\beta}^{\mu} - \frac{1}{2} R \delta_{\alpha\beta}^{\mu\nu} = (1)$$

Hereafter the notations are used

$$\delta_{\alpha\beta}^{\mu\nu} = \delta_{\alpha}^{\mu} \delta_{\beta}^{\nu} - \delta_{\alpha}^{\nu} \delta_{\beta}^{\mu}, \quad g_{\alpha\beta\mu\nu} = g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\nu} g_{\beta\mu}.$$

Professor D. Fairlie kindly informed me of the remarkable Lanczos paper [6], where betweenness relation $R_{\alpha\beta\mu\nu}$ and $*R_{\alpha\beta\mu\nu}$ probably first had been obtained for the euclidean signature space-time.

The next important step is to expand the Riemann tensor into sum of two parts,

$$R_{\alpha\beta\mu\nu} = \frac{1}{2} (R_{\alpha\beta\mu\nu} + *R_{\alpha\beta\mu\nu} + R_{\alpha\beta\mu\nu} - *R_{\alpha\beta\mu\nu}) = R_{\alpha\beta\mu\nu} + S_{\alpha\beta\mu\nu}, \quad (2)$$

where

$$R_{\alpha\beta\mu\nu} = \frac{1}{2} (R_{\alpha\beta\mu\nu} - *R_{\alpha\beta\mu\nu}), \quad S_{\alpha\beta\mu\nu} = \frac{1}{2} (R_{\alpha\beta\mu\nu} + *R_{\alpha\beta\mu\nu}). \quad (3)$$

Now one can represent the tensors $R_{\alpha\beta\mu\nu}$ and $S_{\alpha\beta\mu\nu}$ by Riemann's tensor and Ricci's tensor and scalar

$$R_{\alpha\beta\mu\nu} = R_{\alpha\beta\mu\nu} - \frac{1}{2} (g_{\alpha\mu} R_{\beta\nu} + g_{\beta\nu} R_{\alpha\mu} - g_{\alpha\nu} R_{\beta\mu} - g_{\beta\mu} R_{\alpha\nu} - \frac{1}{2} R g_{\alpha\beta\mu\nu}). \quad (4)$$

$$S_{\alpha\beta\mu\nu} = \frac{1}{2} (g_{\alpha\mu} R_{\beta\nu} + g_{\beta\nu} R_{\alpha\mu} - g_{\alpha\nu} R_{\beta\mu} - g_{\beta\mu} R_{\alpha\nu} - \frac{1}{2} R g_{\alpha\beta\mu\nu}). \quad (5)$$

One must already say something about the tensorial properties. The $S_{\alpha\beta\mu\nu}$ is noteworthy. Note it is expressed by the Ricci tensor and scalar only, not by Riemann's.

The tensor $R_{\alpha\beta\mu\nu}$ should not be confused with the Weyl conformal tensor $C_{\alpha\beta\mu\nu}$

$$R_{\alpha\beta\mu\nu} = C_{\alpha\beta\mu\nu} + \frac{1}{12} R g_{\alpha\beta\mu\nu}.$$

Further, when twice dual conjugating, both $S_{\alpha\beta\mu\nu}$ and $R_{\alpha\beta\mu\nu}$ transform *simply*

$$*S_{\alpha\beta\mu\nu} = +S_{\alpha\beta\mu\nu}, \quad *R_{\alpha\beta\mu\nu} = -R_{\alpha\beta\mu\nu}, \quad (6)$$

i. e., $S_{\alpha\beta\mu\nu}$ and $R_{\alpha\beta\mu\nu}$ are respectively twice selfdual and antiselfdual parts of the curvature tensor.

It makes sense to introduce a new "quantum" number --- d-parity, characterizing behavior of tensors (like curvature one) under twice dual conjugation. For example, $R_{\alpha\beta\mu\nu}$ is odd, and $S_{\alpha\beta\mu\nu}$ --- even under d-parity reflection. Two more examples of d-odd tensors are $g_{\alpha\beta\mu\nu}$ and $E_{\alpha\beta\mu\nu}$.

There are nontrivial equations

$$S_{\alpha\beta\mu\nu} = 0 \quad (7)$$

and

$$R_{\alpha\beta\mu\nu} = 0. \quad (8)$$

These equations have a direct relationship to instantons in nonabelian gauge theories. In particular in the case of $SO(4)$ or $SU(2)$ gauge group, they describe the Belavin-Polyakov-Schwarz-Tyupkin instanton and anti-instanton [1].

Below we shall see the equation ((8)) describes the gravitational instantons.

Some solutions to these equations have been obtained in [5]. For example, the equation ((7)) has a static solution in the metric

$$ds^2 = e^{v(t,r)} dt^2 - e^{\lambda(t,r)} dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (9)$$

Six equations ((7)) with nonvanishing left member reduce to the only second order equation

$$\lambda = -v, \quad v(r) + v^2(r) = \frac{2}{r^2} (1 - e^{-v}).$$

The solution is

$$e^v = 1 + C_1 r^2 + \frac{C_2}{r}. \quad (10)$$

Thus central-symmetric solution to equation ((7)) is static and quite similar to Schwarzschild's [8] except for $C_1 r^2$. It is not without purpose and we'll be back to this as well as to equations ((7)) and ((8)). Below we'll see that for equation ((7)) with vanishing right hand side $C_2 = 0$.

FROM EINSTEIN'S TO GRAVITATIONAL YANG-MILLS' EQUATIONS

Solution to equation ((7)) including the Schwarzschild solution suggests that it is possible to use this equation instead of Einstein's [2]

$$R_{\alpha\mu} - \frac{1}{2} R g_{\alpha\mu} = \Lambda g_{\alpha\mu} + T_{\alpha\mu}, \quad R + T = -4\Lambda \quad (11)$$

although in emptiness. Really, solution to this equation in the metric ((9)) coincide with ((10)), if $C_1 = \Lambda$. Even more so, the tensor $S_{\alpha\beta\mu\nu}$ in the left hand side ((7)) is fully determined by the Ricci tensor.

At this point, we want to call attention to one little drawback to the Einstein equations. Of course, at times, there had been many discussions treating various advantages and disadvantages of these equations although the former are of the overwhelming majority. This one has most likely been discussed

before. The point is that ((11)) are system of differential equations of second order in metric. And the Schwarzschild solution has merely one integration constant. With more detailed examination of Schwarzschild's problem, it turns out that among the equations there are both first order equations and second order. With all that, solutions to first order equations and second order equations are compatible provided that one of the two integration constants is strictly fixed. It is the reason that the Einstein second order equation system solution (the Schwarzschild solution) contains solely one integration constant.

This fact is of course known but completely ignored. We find on this point, Einstein's equations are somewhat inconsistent.

Further heuristically, there will be obtained equation to generalize Einstein's equation ((11)). Then the new equation will be proclaimed as one of the basic equations describing gravity produced by matter. After that, it will be seen the new equation implies the gravitational Yang-Mills equation. Finally, Einstein's equations will be shown to follow from both the new equation and gravitational Yang-Mills'.

First expressing the Ricci tensor from ((11)) and substituting that in ((5)), we can find

$$S_{\alpha\beta\mu\nu} = \Theta_{\alpha\beta\mu\nu}, \quad (12)$$

where

$$\Theta_{\alpha\beta\mu\nu} = \frac{1}{2}(g_{\alpha\mu}T_{\beta\nu} + g_{\beta\nu}T_{\alpha\mu} - g_{\alpha\nu}T_{\beta\mu} - g_{\beta\mu}T_{\alpha\nu} - \frac{1}{2}Tg_{\alpha\beta\mu\nu}). \quad (13)$$

This tensor is built from the metric tensor and the energy-momentum tensor. Later it will be used instead of the energy-momentum tensor. Note that the tensor $\Theta_{\alpha\beta\mu\nu}$ is d-even like $S_{\alpha\beta\mu\nu}$.

Now the important step follows. Let us forget the Einstein equation and instead consider the equation ((12)) as one of the basic equations of gravity generated by matter.

Differentiate covariantly with respect to x^ν the equation ((12))

$$S_{\alpha\beta\mu;\nu}^\nu = \Theta_{\alpha\beta\mu;\nu}^\nu. \quad (14)$$

Having remembered what is $S_{\alpha\beta\mu\nu}$ ((5)), it follows that¹

$$R_{\alpha\beta\mu;\nu}^\nu = 2\Theta_{\alpha\beta\mu;\nu}^\nu \equiv J_{\alpha\beta\mu}. \quad (15)$$

So, the tensor $\Theta_{\alpha\beta\mu\nu}$ determines the matter current or the gravity matter source in the gravitational Yang-Mills equation. The current is not even conserved covariantly since the multiple covariant derivatives don't commute.

Now making contraction over α, μ

$$R_{\beta;\nu}^\nu = 2\Theta_{\beta;\nu}^\nu$$

and taking into account the Bianchi identity and explicit form of the tensor $\Theta_{\alpha\beta\mu\nu}$ we obtain

$$(R + T)^\beta = 0. \quad (16)$$

After integration

$$R + T = -4\Lambda, \quad (17)$$

where -4Λ is the integration constant.

Now we have arrived at the cross-roads. There are two alternatives. One can consider the equation ((12)) as a basic one. Then it implies both equations ((15)) and ((17)).

Otherwise, we can consider the gravitational Yang-Mills equation ((15)) as the basic one. Then we have to take ((12)) as a condition. This second option is preferable.

Once again let's go back to equation ((15)). It is of the form of the Yang-Mills equation. The proposal is to consider it as a basic gravitational equation. And the equality ((17)) is the integral of motion, i. e. a conservation law.

Let's demonstrate that the Einstein equations are implied by the basic equations ((15)) and ((12)). First the conservation law ((17)) is obtained from ((15)). Then contract ((12)) over (β, ν) and eliminate T by means of ((17)). As a result, we have exactly Einstein's equations ((11)). Λ , the constant is obviously interpreted as a *cosmological term* appeared as an integration one!

Thus it is shown that the equations ((12)), ((15)) are equivalent to Einstein's. It is the equations those are the basic gravitational equations. The equation ((15)) is the *basic dynamic* one, and another one ((12)) is a *side condition* which among fields singles out those generated by matter.

Coming back to equations ((12)), or to ((7)) in emptiness, we can see the equations solution in emptiness ((10)) includes two integration constants, one of which apparently associates with Λ . The solution describes (out of the matter distribution) empty constant curvature space with the scale factor $1/\sqrt{\Lambda}$ and with the central-symmetry matter distribution about the point of origin. Let a point mass be at origin. Then with this origin, the metric is given by ((10)), and C_2 is proportional to the mass. As for another constant, it seems to be possible to choose C_1 proportional to Λ . Thus ((10)) is the Schwarzschild static solution in the constant curvature space. We consider it as a manifestation of fact that gravity is nonabelian. The solution ((10)) describes the local geometry in the neighborhood of some spherically symmetric matter distribution. This geometry is determined by both the mass (more precisely, energy-momentum) and Λ . Is that Λ the same in case of any mass or not? In other words, is the cosmological constant Λ *universal*? Assume for a while that it is not the case and Λ is specific for each mass and has an arbitrary value. Then the cubic equation

$$g_{00} = e^\nu = 1 + C_1 r^2 + \frac{C_2}{r} = 0$$

determines the generalized Schwarzschild spheres radii (horizons), number of which is up to three. This would essentially affect the black holes theory.

Classically, the solution ((10)) if you wish could be interpreted as exhibition of asymptotic freedom in gravity.

It specially should be noted that the vacuum solution (in empty space) to the equation ((12)), i. e. ((10)) is nontrivial, as distinct from the Einstein theory. That is, this solution does not just reduce to the Minkowski spacetime. There are both static solutions ((10)) and nonstatic ones with the de Sitter asymptotic solution. For example, for the (closed) Robertson-Walker metric

$ds^2 = dt^2 - a^2(t)(d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\phi^2))$ (18)
the equation ((12)) in emptiness for the scale factor $a(t)$ is of the form

$$a\ddot{a} - \dot{a}^2 - 1 = 0.$$

The vacuum solution is

$$a(t) = a_0 \cosh \frac{t - t_0}{a_0}.$$

Similarly, for the open metric

$$ds^2 = dt^2 - a^2(t)(d\chi^2 + \sinh^2\chi(d\theta^2 + \sin^2\theta d\phi^2))$$
 (19)

the equation for $a(t)$

$$a\ddot{a} - \dot{a}^2 + 1 = 0,$$

has a solution

$$a(t) = a_0 \sinh \frac{t - t_0}{a_0}$$

or

$$a(t) = a_0 \sin \frac{t - t_0}{a_0}.$$

For the latter case $a(t)$ is alternating in sign that seems not to be of physical meaning.

One can use the model equations e. g. to construct cosmological models. That's done. On cursory examination, the Einstein-Friedmann cosmology remains intact. However now the cosmological term seem to be the necessary element of the theory. It should be experimentally measured in the observation cosmology. In the sense, "the dark matter problem" might seem otherwise. Universality of Λ -term in this approach is open to question.

NON-EINSTEINIAN GRAVITY

It is quite clear that the equations ((15)) are more general than the Einstein General Relativity. Namely, any real gravitational field is considered to obey these equations. Of all fields, the Einstein theory extracts the ones to be generated by the matter energy-momentum. Within the theory proposed, the extraction happens by imposing the side condition

((12)). This condition is analogous to self-duality conditions for instantons in the Yang-Mills theory. However it is not the vacuum one. It is possible to treat some other conditions which might extract non-Einsteinian solutions for gravitational fields.

Let us try to discuss possible conditions for gravity. Quite general side condition for equation ((15)) is of the form

$$R_{\alpha\beta\mu\nu} = \kappa R^*_{\alpha\beta\mu\nu} + \varepsilon {}^*R^*_{\alpha\beta}{}^{\mu\nu} + \lambda g_{\alpha\beta\mu\nu} + \zeta E_{\alpha\beta\mu\nu} + 2\Theta_{\alpha\beta\mu\nu}.$$

This is more general than (). This implies the basic equation () fulfilled. In the matter presence $\Theta_{\alpha\beta\mu\nu} \neq 0$, and taking $\varepsilon = -1, \kappa, \lambda, \zeta = 0$, we obtain the equation ((12)). Then the Einstein equation holds and the source conservation takes place. Consequently, the gravitational field equations imply motion equations of matter in the gravitation field generated by the matter.

All that will not occur with alternative set of constants $\varepsilon, \kappa, \lambda, \zeta$. Still admissibility of such a condition is open to question.

In the case of $\Theta_{\alpha\beta\mu\nu} = 0$ we deal with the *vacuum* side conditions.

Vacuum solutions to equation ((15)) could be called gravitational instantons. General representation for instanton is

$$R_{\alpha\beta\mu\nu} = \kappa R^*_{\alpha\beta\mu\nu} + \varepsilon {}^*R^*_{\alpha\beta}{}^{\mu\nu} + \lambda g_{\alpha\beta\mu\nu} + \zeta E_{\alpha\beta\mu\nu}. \quad (20)$$

If the condition holds then gravitational Yang-Mills equation ((13)) will be obeyed. The constants $\kappa, \varepsilon, \lambda, \zeta$ are not quite arbitrary and should be determined.

Alternatively instead of ((20)), one can consider the equation

$$R_{\alpha\beta\mu\nu} = \kappa {}^*R_{\alpha\beta\mu\nu} + \varepsilon {}^*R^*_{\alpha\beta}{}^{\mu\nu} + \lambda g_{\alpha\beta\mu\nu} + \zeta E_{\alpha\beta\mu\nu}.$$

Its solution provides the equation

$$R_{\alpha}{}^{\beta}{}_{\mu\nu;\beta} = 0$$

to be fulfilled.

The analysis of the equation ((20)) is rather complicated so we shall restrict our consideration to particular cases.

We have already discussed the vacuum solutions to the equation ((12)). Here is another possibility: $\varepsilon = 1; \kappa, \zeta = 0$, λ is arbitrary. Then we have the non-Einsteinian equation

$$R_{\alpha\beta\mu\nu} = \lambda g_{\alpha\beta\mu\nu}. \quad (21)$$

Both left and right members of the equation are d-even. Contracting over α, μ and β, ν , we obtain

$$R = 12\lambda.$$

Let's remind ourselves that we have the matter vacuum, hence $T = 0$. It seems correct in cosmology to relate λ to the cosmology term. Generally it is arbitrary and possibly relates to the local vacuum fluctuations of the gravitational fields in Universe.

The equation ((21)) can be solved in the (closed) Robertson-Walker metric. The equation for $a(t)$

$$a\ddot{a} + \dot{a}^2 + 1 = -2\lambda a^2 \quad (22)$$

has a solution

$$a(t) = \sqrt{C_1 \exp(2\sqrt{-\lambda}t) + C_2 \exp(-2\sqrt{-\lambda}t) - \frac{1}{2\lambda}}. \quad (23)$$

It is not analytic in λ . Note a simple constant solution

$$a(t) = a_0 = -\frac{1}{2\lambda}.$$

This solution describes the empty constant positive curvature space. Could not it be called a gravipole? Really it is the same solution as the static solution ((10)) to equation ((7)) in emptiness, i. e. with $C_2 = 0$. It is impossible to pass directly on to $\lambda = 0$ in this metric. However that corresponds to the solution to equation ((21)) as the empty Minkowski space.

In the case $\lambda = 0$, there is a time-dependent solution in the metric ((18))

$$a(t) = a_0 \sqrt{1 - \frac{t - t_0}{a_0}}^2.$$

There are similar solutions in the open Robertson-Walker metric as well. Interestingly, the matter motion (e. g. the test point mass) in the vacuum gravitational fields is already not determined by the field equations but obeyed the geodesic equation.

New approach allows more directly than before to discuss topological effects in gravitation. Really, the conditions ((21)) are “topological”. Projecting ((21)) onto $R^{\alpha\beta\mu\nu}$ (i. e. multiplying and contracting) results in

$$R^{\alpha\beta\mu\nu} R_{\alpha\beta\mu\nu} - 2\lambda R = \kappa R^{\alpha\beta\mu\nu} R^*_{\alpha\beta\mu\nu} + \varepsilon R^{\alpha\beta\mu\nu} *R^*_{\alpha\beta\mu\nu}.$$

After integration, we can see that (in case of convergence) topological numbers can be expressed in terms of invariants quadratic and linear in curva-

ture. Nontrivial topological solutions seem to exist in manifolds with Euclidean signature.

CONCLUSION

So, a new version of gravity is proposed. It is in form and fact the nonabelian Yang-Mills theory of gravitational field with own rich dynamics and nontrivial topology. The theory contains Einstein General Relativity.

It is impossible to avoid a question: what are the Einstein equations? Do they express any conservation law? Or are they any compatibility conditions due to the gauge group peculiarity? It should be specially noted that in the new theory the dynamics is described by the equation ((15)) in presence of the side condition ((12)) and/or others. And the Einstein equations themselves are sequel of this condition and the conservation law ((17)). We have to consider various conditions as well as Einstein’s equations as *constraints*. Perhaps that might change the situation with quantizing gravity.

As for application the theory to astrophysics and cosmology, it is the next job ahead. At once one can say that standard Einstein-Friedmann cosmological model seems not to change. Cosmological term situation may become more definitive. It is not matter of a taste: to work or not to work with that. One must to measure that. Once again one has to say that it is not clear whether Λ is universal.

This new approach has nothing to say as yet about black hole physics. The task in hand is to search for nontrivial topology solutions. It may be time to pass over from chattering about spacetime foam and quantizing gravity [3], [7] to practice.

The theory proposed is natural from the viewpoint the unity of interactions. Gauge invariance and duality are the underlying ideas. Some of these ideas are not yet exhausted in gravity and of interest to apply in the Yang-Mills theory. But this is another topic.

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NOTES

¹ Cosmas Zachos pointed out that (15) can be obtained without references to duality just by contracting the Bianchi identity, then taking into account Einstein’s equation.

REFERENCES

1. Belavin A., Polyakov A., Schwarz A. and Tyupkin I. Pseudoparticle solutions of the Yang-Mills equations // Phys. Lett. 1975. Vol. B59. P. 85.
2. Einstein A. Die Grundlage der allgemeinen Relativitätstheorie // Ann. d. Phys. 1916. Vol. 49. P. 769.
3. Hawking S. Space-Time Foam // Nuclear Phys. 1978. Vol. B144. P. 349.
4. Kibble T. W. Lorentz invariance and the gravitational field // Journ. of Math. Phys. 1961. Vol. 2. P. 212.
5. Koshkarov A. L. On General Relativity Extension. URL: Arxiv-org e-print archive, hep-th/9710038
6. Lanczos C. A remarkable property of the Riemann-Christoffel tensor in four dimensions // Annals of Math. 1938. Vol. 39, 4. P. 842.

7. Misner C., Thorne K. and Wheeler J. Gravitation. San Francisco: W. H. Freeman and Company Limited, 1973. Vol. 1, 2, 3.
8. Schwarzschild K. Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Berlin, 1916. 189 s.
9. Utiyama R. Invariant Theoretical Interpretation of Interaction // Phys. Rev. 1956. Vol. 101. P. 1597.
10. Yang C. N. and Mills R. L. Conservation of Isotopic Spin and Isotopic Gauge Invariance // Phys. Rev. 1954. Vol. 96. P. 191.

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The thorough analysis of the duality properties of the Riemann curvature tensor points to the possibility of the extension of Einstein's General Relativity to a nonabelian Yang-Mills theory. The equations of motion of the theory are the Yang-Mills' equations for the curvature tensor. Einstein's equations (with cosmological term to appear as an integration constant) are contained in the theory proposed. What is new is that now the gravitational field is not exclusively determined by the matter energy-momentum but can possess its own non-Einsteinian dynamics (vacuum fluctuations, self-interaction) which is generally an attribute of a nonabelian gauge field. The gravitational equations proper due to either matter energy-momentum or vacuum fluctuations are side conditions imposed on the Riemann tensor, like self-duality conditions. One of such conditions in the end results in Einstein's equations, other ones are the gravitational instantons equations.

Key words: nonabelian duality, instanton, noneinsteinian gravity, cosmological term

REFERENCES

1. Belavin A., Polyakov A., Schwarz A. and Tyupkin I. Pseudoparticle solutions of the Yang-Mills equations // Phys. Lett. 1975. Vol. B59. P. 85.
2. Einstein A. Die Grundlage der allgemeinen Relativitätstheorie // Ann. d. Phys. 1916. Vol. 49. P. 769.
3. Hawking S. Space-Time Foam // Nuclear Phys. 1978. Vol. B144. P. 349.
4. Kibble T. W. Lorentz invariance and the gravitational field // Journ. of Math. Phys. 1961. Vol. 2. P. 212.
5. Koshkarov A. L. On General Relativity Extension. URL: Arxiv-org e-print archive, hep-th/9710038
6. Lanczos C. A remarkable property of the Riemann-Christoffel tensor in four dimensions // Annals of Math. 1938. Vol. 39, 4. P. 842.
7. Misner C., Thorne K. and Wheeler J. Gravitation. San Francisco: W. H. Freeman and Company Limited, 1973. Vol. 1, 2, 3.
8. Schwarzschild K. Über das Gravitationsfeld eines Massenpunktes nach der Einsteinschen Theorie. Berlin, 1916. 189 s.
9. Utiyama R. Invariant Theoretical Interpretation of Interaction // Phys. Rev. 1956. Vol. 101. P. 1597.
10. Yang C. N. and Mills R. L. Conservation of Isotopic Spin and Isotopic Gauge Invariance // Phys. Rev. 1954. Vol. 96. P. 191.

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