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Article

Evaluation of the fatigue wear probability of pavement using fracture mechanics methods

Artem V. Stepanov[1*]; Alexander N. Petrov[1]; Alexander V. Pitukhin [1]

[1] Petrozavodsk State University, Lenin av. 33, Russia, 185910, Petrozavodsk; E-Mails: stepanov@petrsu.ru (A. S.), sssn20088@yandex.ru (A. P.), pitukhin@petrsu.ru (A. P/)

***** Author to whom correspondence should be addressed; E-Mail: stepanov@petrsu.ru (A. S.)
Tel.: 89535295529.

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Abstract: Improving the quality of pavement and reducing the cost of its maintenance is an important problem. One of the causes of road raveling is fatigue wear of surface materials..The process of materials fatigue wear is described by the linear fracture mechanics methods. The Paris equation describes the rate of fatigue crack growth. On the basis of Paris equation we suggest a method for estimating the fatigue wear probability of the pavement. The statistical linearization method is used for the probabilistic description of the crack growth process. The lifetime distribution law is assumed as lognormal. The example of the calculation of asphalt pavement fatigue wear probability is given in order to test the method. The adequacy of the proposed models is shown. It is established that the variation of the loading and pavement properties significally affect the probability of fatigue wear. Some ways of improving the proposed method are suggested.

Keywords: pavement, fatigue wear, lifetime, wear probability, lognormal distribution, statistical linearization.

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1. Introduction

One of the main types of pavements destruction is fatigue. Fatigue occurs under the action of cyclic loads. It is generally accepted that the fatigue crack growth may be analyzed in terms of linear elastic fracture mechanics by using the stress intensity factor K [1]. The crack growth rate per cycle, dl/dN, can be expressed by the Paris relationship [2]:

$$\frac{dl}{dN} = C(\Delta K)^n, \tag{1}$$

where C and n are empirical parameters; ΔK is the range of stress intensity factor:

$$\Delta K = K_{\max} - K_{\min}.$$

Here, $K_{\max}$ and $K_{\min}$ are, respectively, maximum and minimum values of the stress intensity factor in the cycle, derived from the maximum and minimum stress values. Equivalent expressions may be obtained by using the cycle asymmetry factor (stress ratio) R [1], [3], [4]:

$$\Delta K = Y(l)\sigma_{1max}(1 - R)\sqrt{\pi l}, \tag{2}$$

where $Y(l)$ is a dimentionless geometric factor taking into account the object shape and crack length (semilength) l;

σ_{1max} — is the maximum in the cycle principal applied stress.

Maarten et al [5] using finite element simulation determined the relationship between crack opening displacement, crack length and the stress intensity factor K, and they also determined the Paris[i] parameters C and n of the tested asphalt mixture.

New criteria including energy are suggested to assess fatigue durability in Livnex articles [6] and Maggiore et al. [7].

It should be noted that the parameters are considered as deterministic in most publications.

However it is known that the properties of the asphalt concrete and loading are random.

A number of papers using the probability theory in fracture mechanics are published [3], [4], [8], [9], [10], [11]. Probabilistic and statistical methods in fracture mechanics developed in three main directions [12]. The simplest approach is to extrapolate the past wear date and experience while ignoring explicit mechanical and physical detail of the system. Such a statistical analysis is called the “data-base” approach that is used in [13], [14], [15], [16]. The second approach which deals with probabilistic fracture mechanics is reported elsewhere [8], [10], [17]. In this case the statistical variations of each model parameter are quantified and combined to compute safety as risk. The third approach known as combined analysis utilizes both the data-base and probabilistic fracture mechanics approaches [9]. Widely spread Monte-Carlo method is classified into the second approach. However, the method of statistical linearization, which is proposed by Serensen et al. [18] and is often used in mechanical engineering, is rarely used in road modeling. Therefore, in the present paper we describe a method of the fatigue wear probability estimation of the pavement using statistical linearization and fracture mechanics.

It is also expedient to give an example of calculation and to give primary analysis. The directions of further development of the method are given in the paper.

2. Materials and Methods

We suppose that pavement loading is assigned as a block $\{\sigma_{1i}, w_i\}$ where w_i is the number of cycles at the *i-th* block step; σ_{1i} - is the maximum principal stress on the lower plane of asphalt concrete pavement in the *i-th* step of the loading block. Block size is:

$$w_b = \sum_{i=1}^{m} w_i,$$

where *m* is the number of steps.

Assuming the validity of the linear cumulative damage rule, one can obtain a relation for the number evaluation on crack propagation from the initial defect size l_0 to the final crack size l_c by integration of egn (1) and taking into account egn (2) under the condition *R* = 0:

$$N_p = \frac{w_b}{C\pi^{n/2} \sum_{i=1}^{m} w_i \sigma_{1i}^n} \int_{l_0}^{l_c} \frac{dl}{Y^n(l) l^{n/2}}. \quad (3)$$

Under the assumption of the smallness of l_0 compared with pavement thickness t and t = l_c may be considered $Y(l) = Y = const$.

The equation takes the form after integration:

$$N_p = \frac{2w_b \left(l_c^{\frac{2-n}{2}} - l_0^{\frac{2-n}{2}} \right)}{C(2-n) Y^n \pi^{n/2} \sum_{i=1}^{m} w_i \sigma_{1i}^n}, \quad (4)$$

In the case of irregular loading in accordance with the recommendation of Serensen et al. [18] random variations in the stress distribution function can be described by the relation:

$$\sigma_{1i} = \bar{\sigma}_{1i}\mathcal{E},$$

where $\bar{\sigma}_{1i}$is the mean of the stress level of the i stage of the histogram (block); $\mathcal{E}$ — factor, which takes into account the impact of non-regulated settings (temperature, weather conditions etc…). $\mathcal{E}$ is a normally distributed random variable with a mean value $\bar{\mathcal{E}} = 1$ and coefficient of variation $\nu_{\mathcal{E}} = 0.1$ [18].

Denoting

$$A = \frac{2\left(l_c^{\frac{2-n}{2}} - l_0^{\frac{2-n}{2}}\right)}{(2-n)Y^n\pi^{n/2}},$$

we obtain the equation

$$N_p = \frac{A}{C\mathcal{E}^n \sum_{i=1}^{m} w_i\bar{\sigma}_{1i}^n}. \quad (5)$$

We believe that in addition to the loading on the coating, the properties of pavement materials are random too. So we expect that constant C from the Paris[1] equation is a random variable with mean $\bar{C}$. Then the mean lifetime fatigue crack propagation in accordance with the statistical linearization method [18].

$$\bar{N}_p = \frac{A}{\bar{C}\sum_{i=1}^{m} w_i\bar{\sigma}_{1i}^n}. \quad (6)$$

To determine the variance of lifetime logarithm dependence (S):

$$lgN_p = lgA - lgC - nlg\mathcal{E} - lg\sum_{i=1}^{m} w_i\bar{\sigma}_{1i}^n,$$

Denoting

$$A_1 = lgA - lg\sum_{i=1}^{m} w_i\bar{\sigma}_{1i}^n,$$

we obtain

$$lgN_p = A_1 - lgC - nlg\mathcal{E}. \quad (7)$$

In equation (7) there are two random variables C and $\mathcal{E}$. The variance of the logarithm of lifetime is determined by the statistical linearization method:

$$S_{lgN_p}^2 = \left(\frac{\partial lgN_p}{\partial C}\right)^2_{|\ C=\bar{C}} S_C^2 + \left(\frac{\partial lgN_p}{\partial \mathcal{E}}\right)^2_{|\ \mathcal{E}=\bar{\mathcal{E}}} S_{\mathcal{E}}^2 \tag{8}$$

Where

$$\left(\frac{\partial lgN_P}{\partial C}\right)^2_{|\ C=\bar{C}} S_C^2 = \left(0.434\frac{1}{\bar{C}}\right)^2 S_C^2 = \left(0.434\frac{S_C}{\bar{C}}\right)^2 = 0.188\nu_C^2,$$

$$\left(\frac{\partial lgN_P}{\partial \mathcal{E}}\right)^2_{|\ \mathcal{E}=\bar{\mathcal{E}}} S_{\mathcal{E}}^2 = 0.188n^2\nu_C^2.$$

Or

$$S_{lgN_p}^2 = 0.188(\nu_C^2 + n^2\nu_{\mathcal{E}}^2). \tag{9}$$

We assume that the distribution law of lifetime is lognormal. The lognormal distribution is commonly used for general reliability analysis, for description of cycles distribution till fatigue material strengths and loading variables in for description of probabilistic design [18], [19]. In this case, the lifetime logarithm of the Q wear probability can be determined:

$$lgN_Q = lg\bar{N}_p + U_Q S_{lgN_p}, \tag{10}$$

where U_Q — quantile of the normal distribution for the probability Q.

Crack-like defects l_0 may be different, for example: the depth of the seam in the strengthen cover, the size of the largest fractions of filler etc.

3. Results

To verify the possibility of using Paris equation (1) the comparison of the experimental data with the results of calculation was made. These experimental data for the crack growth rate were obtained by V. L. Trigoni et al. [20] for fine-grained asphalt concrete with a (modulus of elasticity) Young's modulus E = 1500 MPa. In the above publication fatigue crack growth rate is determined in cm/cycle and the stress intensity factor is determined in kg/cm$^{3/2}$. After processing of the experimental data presented in the publication [20] in the thesis [21] A. N. Petrov obtained values of the Paris[1] constant n = 4.11 and C = 7,52*10^{-7}. Similarly, the results of known publication [22] were obtained n = 4.07 and C = 2.62*10^{-7}. The results are shown in the table below.

Table 1 — Comparison of experimental and calculated data

$K_{I\,max}$, kg/cm$^{3/2}$	Crack growth rate dl/dN*10^{-6}, cm/cycle		Relative error, %
	Experiment	Calculation	
33.9	1.52	1.49	2
42.6	3.96	3.81	4
50.2	7.12	7.48	5
58.0	12.75	13.54	6
66.4	23.3	23.62	1
75.3	40.98	39.64	3
85.5	68.68	66.89	3

Analyzing the results presented in Table 1 we come to the conclusion that it is possible to use equation (1) to predict the fatigue failure of asphalt pavement.

The testing of the method produced an example of calculating the wear probability of asphalt pavement due to fatigue. Through the size of crack-life defect$l_0 = 20mm$, the critical crash size $l_c = 100mm$, the constants of the Paris equation n = 4.11 and C = 7,52*10^{-13}, the coefficients of variation $\nu_\varepsilon = 0.3$ and $\nu_C = 0.3$.

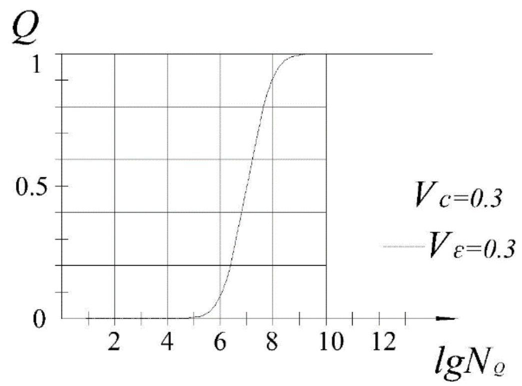

Figure 1. The dependence of fatigue wear from the load and properties of the pavement.

Block loading was determined by calculation of the first principal stresses at the depth of $100mm$ from the impact on coverage of loads from the train Sisu Timber C500 and presented in Table 2.

Table 2 — Blok loading

$\sigma_{1i}, kg/cm^2$	6	7	8
W_i	4	1	2

The calculation results are presented in figure 1. The results of calculations show that with a probability of 10 percent fatigue wear of the coating will occur after $1.15*10^6$ cycles. The mean lifetime is $1.03*10^7$ cycles. The calculations for other values of the coefficients of variation ν_ε and ν_C were also carried out.

For example, if $\nu_\varepsilon = \nu_C = 0.1$ ten percent lifetime is $2.9*10^6$ cycles, which is almost three times different from the previous calculation.

4. Discussion and Conclusions

1. The method for estimating the fatigue wear probability of the pavement is presented. The method is based on the use of Paris equation describing the growth rate of a fatigue crack. For the probabilistic description of the fatigue process the statistical linearization method is used. The distribution of the lifetime was assumed as lognormal.

2. The example of calculation of asphalt pavement is presented in the article. It is shown that the variation of loads and pavement properties significally affect the probability of fatigue wear.

3. Further development of the proposed method may go by taking into account the size of the plastic zone at the crack tip [23] and by using Monte-Carlo method. The possibility of the Paris modified equation use is suggested in the paper [11].

4. In addition, it is desirable to take into account the stiffness and the composition of the base road coating.

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