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B. F. Ivanov

ANALOG OF AN INEQUALITY OF BOHR FOR INTEGRALS OF FUNCTIONS FROM $L^p(R^n)$. I

Abstract. Let $p \in (2, +\infty]$, $n \geq 1$ and $\Delta = (\Delta_1, \ldots, \Delta_n)$, $\Delta_k > 0$, $1 \leq k \leq n$. It is proved that for functions $\gamma(t) \in L^p(R^n)$ spectrum of which is separated from each of n the coordinate hyperplanes on the distance not less than Δ_k, $1 \leq k \leq n$ respectively, the inequality is valid:

$$\left\| \int\limits_{E_t} \gamma(\tau)\, d\tau \right\|_{L^\infty(R^n)} \leq C^n(q) \left[\prod_{k=1}^{n} \frac{1}{\Delta_k^{1/q}} \right] \|\gamma(\tau)\|_{L^p(R^n)},$$

where $t = (t_1, \ldots, t_n) \in R^n$, $E_t = \{\tau \,|\, \tau = (\tau_1, \ldots, \tau_n) \in R^n$, $\tau_j \in [0, t_j]$, if $t_j \geq 0$, and $\tau_j \in [t_j, 0]$, if $t_j < 0$, $1 \leq j \leq n\}$, and the constant $C(q) > 0$, $\dfrac{1}{p} + \dfrac{1}{q} = 1$ does not depend on $\gamma(\tau)$ and vector Δ.

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Let us consider an arbitrary $\Lambda > 0$ and denote by $P(\Lambda)$ the set of all finite trigonometric sums

$$p(t) = \sum_{m=1}^{N} p_m e^{i\lambda_m t},$$

the Fourier exponents of which satisfy the following condition

$$\min_{1 \leq m \leq N} |\lambda_m| \geq \Lambda.$$

H. Bohr announced [1] and proved [2] that for such sums the next inequality, which later ([3–5] and etc.) was named after Bohr, is valid

$$|p(t)| \leq \frac{\pi}{2\Lambda} \|dp(t)/dt\|_{L^\infty(R^1)}, \quad p(t) \in P(\Lambda),$$

There exist several generalizations of this inequality – the details see in the author's paper [6]. The author's generalization was proposed in the article [6] mentioned above. Unlike other authors results in [6] the Bohr inequality was considered as the inequality which gives the estimation of the integral of the function via the norm of this function for some subclasses of the $L^p(R^n)$ space for $p \in (1, 2]$. The proof of the main results in [6] were significantly based on Hausdorff–Young inequality which did not permit to extend the obtained results to the case $p > 2$. In present paper the case $p \in (2, +\infty]$ is considered.

Let $p \in (2, +\infty]$, $n \geq 1$, $\Delta = (\Delta_1, \Delta_2, \ldots, \Delta_n)$ is any vector with positive coordinates and $t = (t_1, t_2, \ldots, t_n) \in R^n$.

Let us introduce the following designations:

1) $Q(\Delta) = \bigcup\limits_{k=1}^{n} \{y \,|\, y = (y_1, y_2, \ldots, y_n) \in R^n,\ |y_k| < \Delta_k\}$, i. e. $Q(\Delta)$ – "cross" origin neighborhood in R^n;
2) $\Gamma(R^n \setminus Q(\Delta), p)$ – the set of all functions $\gamma(t) \in L^p(R^n)$, the Fourier transformations supporters of which are in $R^n \setminus Q(\Delta)$;
3) $E_t = \{\tau \,|\, \tau = (\tau_1, \tau_2, \ldots, \tau_n) \in R^n,\ \tau_j \in [0, t_j]$, if $t_j \geq 0$, and $\tau_j \in [t_j, 0]$, if $t_j < 0,\ 1 \leq j \leq n\}$ – is the parallelepiped in R^n.

The main assertion of this paper is the theorem 3.2, section 3, which is the following.

Theorem 3.2. *Let* $n \geq 1$, $p \in (2, +\infty]$ *and* $\Delta = (\Delta_1, \ldots, \Delta_n)$, $\Delta_k > 0$, $1 \leq k \leq n$. *Then for any function* $\gamma(\tau) \in \Gamma(R^n \setminus Q(\Delta), p)$ *the next inequality is fulfilled:*

$$\left\| \int\limits_{E_t} \gamma(\tau) d\tau \right\|_{L^\infty(R^n)} \leq C^n(q) \left[\prod_{k=1}^{n} \frac{1}{\Delta_k^{1/q}} \right] \|\gamma(\tau)\|_{L^p(R^n)}, \tag{0.1}$$

where $\dfrac{1}{p} + \dfrac{1}{q} = 1$, *and the constant* $C(q) > 0$ *does not depend on* $\gamma(\tau)$ *and vector* Δ.

Let us note that if $n = 1$ and $p = +\infty$ then the inequality (0.1) may be obtained from [4] and [5].

The assertion of the theorem 3.2 for $n = 1$ was essentially employed by the author for the construction of the frequency criteria of the boundedness and smoothness in Frechet sense with respect to the parameters of the ordinary differential equation systems solutions [7], and also for the boundedness of the nonlinear differential equations solutions [8].

The paper consists of the introduction and the three sections the first two of which are preliminary and the third contains theorem 3.2.

The proposed article contains §1, §2 and presents the first part of the paper; the main second part contains §3 and is prepared for publication.

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§1. Designations and Lemma on the Fourier Transformation Support of Several Functions Product

The section begins with the notation form choice of the Fourier transformation and the summary of several notations of the standard formulas following from such form. Thereafter the lemma 1.1 on the Fourier transformation support of several functions product in the case which is not suited for the direct employment of general theorems (for instance, [9, ch. 1, §5], [10, ch. 2, §7]) on the existence of resultant and on the support is proved. The assertion of lemma 1.1 will be used in §3.

Let $n \geq 1$. Following [11, p. 77], let us denote the Fourier transformation of the function by $u(t) \in L^1(R^n)$ as $\hat{u}(y)$, where $y \in R^n$, but following [12, p. 425], let us choose $\hat{u}(y)$ as

$$\hat{u}(y) = \int\limits_{R^n} e^{-i(y,t)} u(t) dt.$$

The reciprocal Fourier transformation of the function $v(y) \in L^1(R^n)$, also following [11, p. 77], let us designate as $\tilde{v}(t)$, $t \in R^n$, where $\tilde{v}(t)$ according [12, p. 427] has the form:

$$\tilde{v}(t) = \left(\frac{1}{2\pi}\right)^n \int\limits_{R^n} e^{i(y,t)} v(y) dy.$$

Following [11, p. 73], [9, p. 31], let us denote the space of infinitely differentiable rapidly decreasing at infinity functions by $S(R^n)$, and [11, p. 77] $S'(R^n)$ — the space of slowly increasing distributions, i. e. [10,

p. 149], the space of slow growth distributions, i. e. the space of linear continuous functionals on $S(R^n)$. According to [11, p. 73], [10, p. 15] let us denote the space of finite infinitely differentiable functions on R^n by $D(R^n)$, and the space of linear continuous functionals on $D(R^n)$ by $D'(R^n)$.

Let us associate with each complex valued locally integrable function $\gamma(t)$, $t \in R^n$, the functional [9, p. 30, p. 32]

$$(\gamma, \varphi) = \int\limits_{R^n} \overline{\gamma(t)} \varphi(t) dt, \quad \varphi(t) \in D(R^n).$$

The distributions from $D'(R^n)$, generated by locally integrable functions, are called [11, p. 75] the regular functions. Since $D(R^n)$ is densely embedded in $S(R^n)$ then $S'(R^n) \subset D'(R^n)$. As is known [11, p. 77], for example, the functionals generated by functions $\gamma(t) \in L^p(R^n)$, $p \geq 1$, are the slowly increasing distributions.

The linear continuous functional on $S(R^n)$, designated as $\hat{\gamma}(y)$ and defined (with regard to the choice of the definition for (γ, φ) and the form of Fourier transformation) as

$$(\hat{\gamma}, \hat{\varphi}) = (2\pi)^n (\gamma, \varphi)$$

is called the slowly increasing function Fourier transformation.

According to the introduced designations we obtain the known formulas (see, for instance, [10, ch. II, §9]) in the following form:

$$\left.\begin{aligned} \{\hat{\gamma}_1(y) * \hat{\gamma}_2(y)\}\tilde{}(t) &= (2\pi)^n \gamma_1(t) \cdot \gamma_2(t) \\ \{\hat{\gamma}_1(y) \cdot \hat{\gamma}_2(y)\}\tilde{}(t) &= \gamma_1(t) * \gamma_2(t) \end{aligned}\right\} \tag{1.1}$$

Let $n \geq 2$, $1 \leq k < n$ and $\Delta_{k+1}, \ldots, \Delta_n > 0$. Let us designate $G(\Delta_{k+1}, \ldots, \Delta_n) = \bigcup\limits_{\beta=k+1}^{n} \{y \,|\, y = (y_1, \ldots, y_n) \in R^n, \, |y_\beta| < \Delta_\beta\}$.

If $n - k = 1$ then $G(\Delta_n)$ is the direct product of R^{n-1} by the interval $(-\Delta_n, \Delta_n)$. If $n - k > 1$ then $G(\Delta_{k+1}, \ldots, \Delta_n)$ is the direct product of R^k by "cross" neighborhood of zero in $R^{n-k} = \{y \,|\, y = (y_{k+1}, \ldots, y_n),$ $y_j \in R^1, \, k+1 \leq j \leq n\}$.

Lemma 1.1. *Let $n \geq 2$, $1 \leq k < n$, $p \in [1, +\infty]$, the functions $g_\lambda(\theta) \in$ $\in L^q(R^1)$, $\frac{1}{p} + \frac{1}{q} = 1$, $1 \leq \lambda \leq k$, $\gamma(\tau) \in L^p(R^n)$, $\Delta = (\Delta_1, \ldots, \Delta_n)$ – the* *vector with positive coordinates and* $\operatorname{supp}\hat{\gamma}(y)\cap Q(\Delta)=\varnothing$. *Then*

$$\left(\operatorname{supp}\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}\,(y)\right)\bigcap$$

$$\bigcap G(\Delta_{k+1},\dots,\Delta_n)=\varnothing.$$

Proof. Let us show that for any test function $\hat{\varphi}(y)\in S(R^n)$, such that

$$\operatorname{supp}\ \hat{\varphi}(y)\subset G(\Delta_{k+1},\dots,\Delta_n) \tag{1.2}$$

the following equality is fulfilled

$$\left(\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}\,(y),\hat{\varphi}(y)\right)=0,$$

which implies the lemma assertion.

Let us take the arbitrary test function $\hat{\varphi}(y)\in S(R^n)$ such that

$$\operatorname{supp}\hat{\varphi}(y)\subset G(\Delta_{k+1},\dots,\Delta_n),$$

and construct the sequence $\{\varphi_m(y)\}_{m=1}^{\infty}$ of the test finite functions converging to $\hat{\varphi}(y)$ in the sense of convergence in the space $S(R^n)$ and satisfying for each $m=1,2\dots$ the following conditions:

1) $\operatorname{supp}\ \varphi_m(y)\subseteq\operatorname{supp}\ \hat{\varphi}(y),$ (1.3)

2) $\operatorname{supp}\ \varphi_m(y)\subset G(\Delta_{k+1},\dots,\Delta_n)\cap\{y\,|\,y=(y_1,\dots,y_n),\ |y_j|<3m,\ 1\le j\le n\}.$ (1.4)

Following [9, p. 32], let us denote the infinitely differentiable function which is unit valued in the cube $\{y\,|\,y=(y_1,\dots,y_n),\ |y_j|\le 1,\ 1\le j\le n\}$, and zero valued outside the cube $\{y\,|\,y=(y_1,\dots,y_n),\ |y_j|\le 2,\ 1\le j\le n\}$ by $e_1(y)$, $y\in R^n$ and set

$$\varphi_m(y)=\hat{\varphi}(y)\cdot e_m(y),\quad e_m(y)=e_1\left(\frac{1}{m}\,y\right),\ m=2,3,\dots$$

Then:

$$\operatorname{supp}\ \varphi_m(y)\subseteq\operatorname{supp}\ \hat{\varphi}(y)\cap\operatorname{supp}\ e_1\left(\frac{1}{m}\,y\right). \tag{1.5}$$

From (1.5) we obtain:
1) the assertion (1.3),
2) supp $\varphi_m(y) \subseteq$ supp $e_1\left(\frac{1}{m}\,y\right) = \{y\,|\,y = (y_1, \dots, y_n),\ |y_j| \le 2m,$
$1 \le j \le n\} \subset \{y\,|\,y = (y_1, \dots, y_n),\ |y_j| < 3m,\ 1 \le j \le n\}.$ (1.6)

Since according assumption supp $\hat{\varphi}(y) \subset G(\Delta_{k+1}, \dots, \Delta_n)$, then (1.4) follows from (1.3) and (1.6). Besides, the definition of the function $e_m(y)$ implies that $\varphi_m(y) = \hat{\varphi}(y)$ in the cube $\{y\,|\,y = (y_1, \dots, y_n),\ |y_j| \le m,\ 1 \le$ $\le j \le n\}$.

Let $r = (r_1, r_2, \dots, r_n)$ be any vector with nonnegative integer coordinates. Then as $m \to \infty$:

$$\frac{\partial^{r_1+r_2+\cdots+r_n}}{\partial y_1^{r_1}\partial y_2^{r_2}\dots\partial y_n^{r_n}}\,\varphi_m(y_1, y_2, \dots, y_n) = D^r\left\{\hat{\varphi}(y)\,e_1\left(\frac{1}{m}\,y\right)\right\} =$$

$$= \{D^r\hat{\varphi}(y)\}\,e_1\left(\frac{1}{m}\,y\right) + \frac{O(1)}{m}.$$

Thus we obtain:
1) the $\varphi_m(y)$ derivatives of any order in any bounded domain uniformly converge to corresponding derivative of the function $\hat{\varphi}(y)$ as $m \to \infty$;
2) since we may point out $U(0,b)$ — the ball of radius $b > 0$ such that supp $\hat{\varphi}(y) \subset U(0,b)$, then (1.3) implies that supp $\varphi_m(y) \subset U(0,b)$ for any $m = 1, 2, \dots$ and consequently for any $k = (k_1, k_2, \dots, k_n)$, $r = (r_1, r_2, \dots, r_n)$, where $k_1, k_2, \dots, k_n,\ r_1, r_2, \dots, r_n$ are the nonnegative integer numbers:

$$|y^k D^r\varphi_m(y)| = \left|y_1^{k_1}y_2^{k_2}\dots y_n^{k_n}\,\frac{\partial^{r_1+r_2+\cdots+r_n}\varphi_m(y)}{\partial y_1^{r_1}\partial y_2^{r_2}\dots\partial y_n^{r_n}}\right| <$$

$$< b^{k_1+k_2+\ldots+k_n}|D^r\varphi_m(y)| \le C_{kr},$$

moreover the constants C_{kr} may be chosen independent of m.

Thus the sequence $\varphi_1(y), \varphi_2(y), \dots, \varphi_m(y), \dots$ converges to $\hat{\varphi}(y)$ in the sense of convergence in $S(R^n)$.

Further since the inclusion (1.4) is strict and the set supp $\varphi_m(y)$ is closed and bounded, then at each $m = 1, 2, \dots$ there exists the set of $n-k$ positive numbers $\rho_{k+1}(m), \rho_{k+2}(m), \dots, \rho_n(m)$ such that $\Delta_j > 3\rho_j(m) >$ > 0, $k+1 \le j \le n$ and

$$\text{supp}\ \varphi_m(y) \subset G(\Delta_{k+1} - 3\rho_{k+1}(m), \dots, \Delta_n - 3\rho_n(m))\cap$$

$$\cap\{y\,|\,y=(y_1,\ldots,y_n),\ |y_j|<3m,\ 1\le j\le n\}. \qquad (1.7)$$

Thus for any test function $\hat\varphi(y)\in S(R^n)$, satisfying (1.2) we have:

$$\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}\,(y),\hat\varphi(y)\right)=$$

$$=\lim_{m\to\infty}\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}\,(y),\varphi_m(y)\right), \qquad (1.8)$$

where $\varphi_m(y)$, $m=1,2,\ldots$ satisfies (1.7).

In the equality (1.8) the set supp $\varphi_m(y)$ is contained, according (1.7), in the set of some complex form which is the intersection of the cube in R^n with the edge $6m$ and the set $G(\Delta_{k+1}-3\rho_{k+1}(m),\ldots,\Delta_n-3\rho_n(m))$. For each $m=1,2,\ldots$ let us transform the expression, which is under the integral in the right-hand side of (1.8), changing the function $\varphi_m(y)$ by the sum of functions the support of which is contained in the set more simple in some sense, namely in the parallelepiped which is the direct product of the cube in R^{n-1} with the edge $6m$ by some small interval specified for each parallelepiped.

Let $a,b>0$, $m\ge 1$ and k be from the lemma assumption. Let us introduce some designations:

1) $\eta(\theta,a,b)$, $\theta\in R^1$ is the auxiliary infinitely differentiable function, satisfying the following conditions: $\eta(\theta,a,b)=1$, if $|\theta|\le a$, and $\eta(\theta,a,b)=0$, if $|\theta|\ge a+b$;

2) $\varphi_m(k+1,y)=\varphi_m(y)\cdot\eta(y_{k+1},\Delta_{k+1}-3\rho_{k+1}(m),\rho_{k+1}(m))$;

3) $\varphi_m(k+l,y)=[\varphi_m(y)-\varphi_m(k+1,y)-\ldots-\varphi_m(k+l-1,y)]\,\eta(y_{k+l},\Delta_{k+l}-$ $-3\rho_{k+l}(m),\rho_{k+l}(m))$, $2\le l\le n-1$;

4) $\varphi_m(n,y)=\varphi_m(y)-\varphi_m(k+1,y)-\ldots-\varphi_m(n-1,y)$.

Then:

$$\varphi_m(y)=\varphi_m(k+1,y)+\varphi_m(k+2,y)+\ldots+\varphi_m(n-1,y)+\varphi_m(n,y). \qquad (1.9)$$

Let us note some properties of the functions $\varphi_m(k+l,y)$, $1\le l\le n$. The definition implies that the function $\eta(y_{k+1},\Delta_{k+1}-3\rho_{k+1}(m),\rho_{k+1}(m))$ is infinitely differentiable with respect to $y_{k+1}\in R^1$, equals 1 for $|y_{k+1}|\le$ $\le\Delta_{k+1}-3\rho_{k+1}(m)$ and equals zero for $|y_{k+1}|\ge\Delta_{k+1}-2\rho_{k+1}(m)$. Hence the function $\varphi_m(k+1,y)$ is infinitely differentiable and $\varphi_m(k+1,y)=$ $=\varphi_m(y)$, when

$$y\in\text{supp }\varphi_m(y)\cap\{y\,|\,y=(y_1,\ldots,y_n),|y_{k+1}|\le\Delta_{k+1}-3\rho_{k+1}(m)\}.$$

Besides supp $\varphi_m(k+1,y) \subseteq$ supp $\varphi_m(y) \cap \{y \,|\, y=(y_1,\dots,y_n),\ |y_{k+1}| < < \Delta_{k+1} - 2\rho_{k+1}(m)\}$.

Thus (1.6) implies that supp $\varphi_m(k+1,y) \subset \{y \,|\, y=(y_1,\dots,y_n),\ |y_i| < 3m,\ 1 \le i \le n,\ i \ne k+1,\ |y_{k+1}| < \Delta_{k+1} - 2\rho_{k+1}(m)\}$.

Further for $l = 2,\dots,n$:
the functions $\varphi_m(k+l,y)$ are, evidently, infinitely differentiable and

$$\text{supp } \varphi_m(k+l,y) \subset \{y \,|\, y=(y_1,\dots,y_n),\ |y_i| < 3m,\ 1 \le i \le n,$$
$$i \ne k+l,\ |y_{k+l}| < \Delta_{k+l} - 2\rho_{k+l}(m)\}, \quad 2 \le l \le n-1; \tag{1.10}$$

$\varphi_m(n,y)$ is infinitely differentiable and

$$\text{supp } \varphi_m(n,y) \subset \{y \,|\, y=(y_1,\dots,y_n), |y_i| < 3m,\ 1 \le i \le n,\ i \ne n,$$
$$|y_n| < \Delta_n - 3\rho_n(m)\}.$$

Hence (1.8) and (1.9) implies that

$$\left(\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right\}\hat{}(y), \hat{\varphi}(y)\right) =$$
$$= \sum_{j=k+1}^{n} \lim_{m\to\infty}\left(\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right\}\hat{}(y), \varphi_m(j,y)\right), \tag{1.11}$$

if each of limits exists.

Let us show that for each $k+1 \le j \le n$ the expression under the sign of limit in the right-hand side of (1.11) equals zero. We can take any natural numbers $m \ge 1$, $j \in \{k+1,\dots,n\}$, the function $\varphi_m(j,n)$ and, following the reasoning from [9, p. 134], let us at first construct the sequence $\{\varphi_m(l,j,y)\}_{l=1}^{\infty}$ of the test functions from $D(R^n)$, converging to $\varphi_m(j,n)$ and such that each function $\varphi_m(l,j,y)$ is the sum of the functions from $D(R^n)$, which are the functions with the separated variables $y_1, y_2, \dots, y_n$.

The Weierstrass theorem (see, for [13, p. 348–349]) implies that for any natural number l we may point out such polynomial $P_{N(l)}(y)$ of some degree $N(l)$, that for each vector $r=(r_1,\dots,r_n)$ with nonnegative integer coordinates satisfying the following condition $0 \le |r| \le l$, where $|r| =$

$= r_1 + \ldots + r_n$, in the cube $\Pi_0 = \{y \,|\, y = (y_1, \ldots, y_n),\ |y_s| \le 4m,\ 1 \le$ $\le s \le n\}$ the following inequality will be fulfilled

$$|D^r \varphi_m(j, y) - D^r P_{N(l)}(y)| < \frac{1}{l}. \tag{1.12}$$

Let us designate:

1) $P_{N(l)}(y) = \sum\limits_{|\nu|=0}^{N(l)} a_{\nu_1\nu_2\ldots\nu_n} y_1^{\nu_1} y_2^{\nu_2} \ldots y_n^{\nu_n}$, where $\nu = (\nu_1, \nu_2, \ldots, \nu_n)$ – the vector with nonnegative integer coordinates, $|\nu| = \nu_1 + \nu_2 + \ldots + \nu_n$ and $a_{\nu_1\nu_2\ldots\nu_n}$ – the constants;

2) $\varphi_m(l, j, y) = P_{N(l)}(y) \left\{ \prod\limits_{\substack{s=1 \\ s \ne j}}^{n} \eta(y_s, 3m, m) \right\} \times \eta(y_j, \Delta_j - 2\rho_j(m), \rho_j(m)).$

(1.13)

Then $\varphi_m(l, j, y)$ is the sum of products of the functions depending only on one of the variables $y_1, y_2, \ldots \ldots, y_n$:

$$\varphi_m(l, j, y) = \sum_{|\nu|=0}^{N(l)} a_{\nu_1\nu_2\ldots\nu_n} \left\{ \prod_{\substack{s=1 \\ s \ne j}}^{n} y_s^{\nu_s} \eta(y_s, 3m, m) \right\} \times$$

$$\times y_j^{\nu_j} \eta(y_j, \Delta_j - 2\rho_j(m), \rho_j(m)) =$$

$$= \sum_{|\nu|=0}^{N(l)} a_{\nu_1\nu_2\ldots\nu_n} \left\{ \prod_{\substack{s=1 \\ s \ne j}}^{n} A_1(\nu_s, y_s) \right\} A_2(\nu_j, y_j), \tag{1.14}$$

where

$$\left. \begin{array}{l} A_1(\nu_s, y_s) = y_s^{\nu_s} \eta(y_s, 3m, m), \quad s \ne j \\ A_2(\nu_j, y_j) = y_j^{\nu_j} \eta(y_j, \Delta_j - 2\rho_j(m), \rho_j(m)), \quad s = j \end{array} \right\}. \tag{1.15}$$

Let us take any $m \ge 1$, $j \in \{k+1, \ldots, n\}$, the function $\varphi_m(j, y)$ and show that the sequence

$$\psi_m(l, j, y) = \varphi_m(l, j, y) - \varphi_m(j, y), \qquad l = 1, 2, \ldots$$

converges to zero in the sense of convergence in $D(R^n)$.

According to the definition (see, for instance, [10, p. 85]), the sequence $\psi_m(1,j,y)$, $\psi_m(2,j,y), \ldots,$ $\psi_m(l,j,y), \ldots$ converges to zero if:
1) there exists the ball $U(0,R)$ of radius $R>0$ with the center at origin such that: supp $\psi_m(l,j,y) \subset U(0,R)$, $l=1,2,\ldots$
2) for each $r=(r_1,\ldots,r_n)$, where r_i, $1\le i\le n$ are the integer nonnegative numbers, the sequence $D^r\psi_m(l,j,y)$ converges uniformly to zero as $l\to\to+\infty$.

Let us denote

$$\Pi_{1j}=\{y\,|\,y=(y_1,\ldots,y_n),\ |y_s|\le 4m,\ 1\le s\le n,\ s\ne j,\ |y_j|\le\Delta_j-\rho_j(m)\},$$

$$\Pi_{2j}=\{y\,|\,y=(y_1,\ldots,y_n),\ |y_s|\le 3m,\ 1\le s\le n,\ s\ne j,\ |y_j|\le\Delta_j-2\rho_j(m)\}.$$

The first condition from the convergence definition is evidently true because according to (1.13):

$$\text{supp}\ \varphi_m(l,j,y)\subseteq\Pi_{1j}\subset\Pi_{0j}\subset U(0,4m\sqrt{n})$$

and according to (1.10):

$$\text{supp}\ \varphi_m(j,y)\subset\Pi_{2j}\subset U(0,4m\sqrt{n}).$$

Now we verify the second condition. Let $y\in\Pi_{2j}$, then according to the definition of the function $\varphi_m(l,j,y)$, from (1.13) we obtain:

$$\varphi_m(l,j,y)=P_{N(l)}(y)\left\{\prod_{\substack{s=1\\ s\ne j}}^{n}\eta(y_s,3m,m)\right\}\times$$

$$\times\eta(y_j,\Delta_j-2\rho_j(m),\rho_j(m))=P_{N(l)}(y),$$

because $\eta(y_s,3m,m)=1$ for $|y_s|\le 3m$, $1\le s\le n$, $s\ne j$ and $\eta(y_j,\Delta_j-{}-2\rho_j(m),\rho_j(m))=1$ for $|y_j|\le\Delta_j-2\rho_j(m)$.

Thus according to (1.12) for $y\in\Pi_{2j}$ the following inequality is true

$$|D^r\psi_m(l,j,y)|=|D^r[\varphi_m(l,j,y)-\varphi_m(j,y)]|=$$

$$=\left|D^r[P_{N(l)}(y)-\varphi_m(j,y)]\right|<\frac{1}{l}.$$

Let $y \in \overline{\Pi_{1j} \setminus \Pi_{2j}}$, then $\varphi_m(j, y) = 0$ and hence for any $l = 1, 2, \ldots$ according to (1.13)

$$D^r \psi_m(l, j, y) = D^r \varphi_m(l, j, y) =$$

$$= D^r \left\{ P_{N(l)}(y) \left[\prod_{\substack{s=1 \\ s \neq j}}^{n} \eta(y_s, 3m, m) \right] \eta(y_j, \Delta_j - 2\rho_j(m), \rho_j(m)) \right\} =$$

$$= D^r \{P_{N(l)}(y) \cdot Y(m, j, y)\},$$

where $Y(m, j, y) = \left[\prod_{\substack{s=1 \\ s \neq j}}^{n} \eta(y_s, 3m, m) \right] \eta(y_j, \Delta_j - 2\rho_j(m), \rho_j(m))$.

According to the formula of product differentiation the function

$$\frac{\partial^{r_1 + r_2 + \ldots + r_n}}{\partial y_1^{r_1} \partial y_2^{r_2} \cdots \partial y_n^{r_n}} \left\{ P_{N(l)}(y) \cdot Y(m, j, y) \right\}$$

is the sum $2^{|r|}$ of the following addends

$$\frac{\partial^{a_1 + a_2 + \ldots + a_n}}{\partial y_1^{a_1} \partial y_2^{a_2} \cdots \partial y_n^{a_n}} P_{N(l)}(y) \cdot \frac{\partial^{b_1 + b_2 + \ldots + b_n}}{\partial y_1^{b_1} \partial y_2^{b_2} \cdots \partial y_n^{b_n}} Y(m, j, y),$$

where $a_1 + b_1 = r_1,\ a_2 + b_2 = r_2,\ \ldots,\ a_n + b_n = r_n$.

Thus for $y \in \overline{\Pi_{1j} \setminus \Pi_{2j}}$ we have

$$|D^r \psi_m(l, j, y)| \leq 2^{|r|} \max_{0 \leq |a| \leq |r|} \left\| \frac{\partial^{a_1 + a_2 + \ldots + a_n}}{\partial y_1^{a_1} \partial y_2^{a_2} \cdots \partial y_n^{a_n}} P_{N(l)}(y) \right\|_{L^\infty(\overline{\Pi_{1j} \setminus \Pi_{2j}})} \times$$

$$\times \max_{0 \leq |b| \leq |r|} \left\| \frac{\partial^{b_1 + b_2 + \ldots + b_n}}{\partial y_1^{b_1} \partial y_2^{b_2} \cdots \partial y_n^{b_n}} Y(m, j, y) \right\|_{L^\infty(\overline{\Pi_{1j} \setminus \Pi_{2j}})},$$

where $|a| = a_1 + a_2 + \ldots + a_n$, $|b| = b_1 + b_2 + \ldots + b_n$, $|r| = |a| + |b|$.

For $l \geq |r|$ according to (1.12) the inequality will be true

$$|D^r \psi_m(l, j, y)| \leq 2^{|r|} \frac{1}{l} \max_{0 \leq |b| \leq |r|} \left\| \frac{\partial^{|b|}}{\partial y_1^{b_1} \cdots \partial y_n^{b_n}} Y(m, j, y) \right\|_{L^\infty(\overline{\Pi_{1j} \setminus \Pi_{2j}})},$$

and because the partial derivatives $\dfrac{\partial^{|b|}}{\partial y_1^{b_1}\cdots\partial y_n^{b_n}}Y(m,j,y)$ are bounded on $\overline{\Pi_{1j}\setminus\Pi_{2j}}$ and independent of l, then as $l\to\infty$:

$$\|D^r\psi_m(l,j,y)\|_{L^\infty(\overline{\Pi_{1j}\setminus\Pi_{2j}})}\to 0,$$

and therefore the sequence $\psi_m(1,j,y)$, $\psi_m(2,j,y)$, $\ldots$ $\ldots,\psi_m(l,j,y),\ldots$ converges to zero in the sense of convergence in $D(R^n)$.

Thus according to the definition of the convergence in $D(R^n)$, the sequence $\varphi_m(1,j,y)$, $\varphi_m(2,j,n)$, $\ldots$, $\varphi_m(l,j,n)$, $\ldots$ converges to the function $\varphi_m(j,n)$ and according to (1.11):

$$\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}(y),\hat{\varphi}(y)\right)=$$

$$=\sum_{j=k+1}^{n}\lim_{m\to+\infty}\lim_{l\to+\infty}(\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\times$$

$$\times\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}(y),\varphi_m(l,j,y)\right). \tag{1.16}$$

For each $l=1,2,\ldots$ according to (1.14) we obtain:

$$\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}(y),\varphi_m(l,j,y)\right)=$$

$$=\sum_{|r|=0}^{N(l)}\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}(y_1,\ldots,y_n),\right.$$

$$\left.a_{r_1\ldots r_n}\left\{\prod_{\substack{s=1\\ s\neq j}}^{n}A_1(r_s,y_s)\right\}A_2(r_j,y_j)\right). \tag{1.17}$$

Assertion 1.1.1. *Let* $n\geq 2$, $k+1\leq j\leq n$ *and* $l\geq 1$. *Then according to the designations and assumptions introduced above:*

$$\left(\left\{\gamma(\tau_1,\ldots,\tau_k,\tau_{k+1},\ldots,\tau_n)\prod_{\lambda=1}^{k}g_\lambda(\tau_\lambda)\right\}\hat{}(y),\right.$$

$$\left\{\prod_{\substack{s=1\\s\neq j}}^{n} A_1(r_s,y_s)\right\} A_2(r_j,y_j)\Bigg) = 0. \qquad (1.18)$$

The proof of assertion 1.1.1. Changing, if it is necessary, the numeration of the variables $y_1, y_2, \ldots, y_n$, we may without the loss of generality suppose that in the equality (1.18): $j = n$. According to the assumption

$$\varnothing = (\text{supp }\ \hat{\gamma}(y)) \cap Q(\Delta),$$

then

$$\varnothing = (\text{supp }\ \hat{\gamma}(y)) \cap \{y \,|\, y = (y_1,\ldots,y_n),\ |y_n| < \Delta_n\} =$$
$$= \text{supp }\ \hat{\gamma}(y) \cap G(\Delta_n).$$

Let $\widehat{\psi}_1(y_1,\ldots,y_{n-1})$ be the arbitrary test function from $S(R^{n-1})$ and $\widehat{\psi}(y) = \widehat{\psi}_1(y_1,\ldots,y_{n-1})A_2(r_n,y_n)$, where, as it is evidently follows from (1.15), $A_2(r_n,y_n)$ is the test function. Then $\widehat{\psi}(y) \in S(R^n)$, supp $\widehat{\psi}(y) \subset$ $\subset G(\Delta_n)$ and

$$\psi(\tau) = \left(\frac{1}{2\pi}\right)^n \int\limits_{R^n} e^{i(y,\tau)}\widehat{\psi}(y)dy = \psi_1(\tau_1,\ldots,\tau_{n-1})\widetilde{A}_2(r_n,\tau_n),$$

where $\tau = (\tau_1,\ldots,\tau_n)$, $y = (y_1,\ldots,y_n)$.

Hence

$$0 = \left(\hat{\gamma}(y), \widehat{\psi}(y)\right) = \left(\frac{1}{2\pi}\right)^n \int\limits_{R^n} \overline{\gamma(\tau_1,\ldots,\tau_n)}\,\psi_1(\tau_1,\ldots,\tau_{n-1})\times$$

$$\times\widetilde{A}_2(r_n,\tau_n)d\tau_1\ldots d\tau_{n-1}d\tau_n = \left(\frac{1}{2\pi}\right)^{n-1} \int\limits_{R^{n-1}} \psi_1(\tau_1,\ldots,\tau_{n-1})\times$$

$$\times\left\{\frac{1}{2\pi}\int\limits_{R^1} \overline{\gamma(\tau_1,\ldots,\tau_{n-1},\tau_n)}\,\widetilde{A}_2(r_n,\tau_n)d\tau_n\right\} d\tau_1\ldots d\tau_{n-1}. \qquad (1.19)$$

Since $\psi_1(\tau_1,\dots,\tau_{n-1}) \in S(R^{n-1})$ is the arbitrary test function then from the equality to zero of the integral in the right-hand side of (1.19), according to Du Bois-Reymond lemma [10, p. 95] we obtain that for almost all the points $(\tau_1,\dots,\tau_{n-1}) \in R^{n-1}$ the following equality is true

$$0 = \frac{1}{2\pi}\int\limits_{R^1} \overline{\gamma(\tau_1,\dots,\tau_{n-1},\tau_n)}\,\widetilde{A}_2(r_n,\tau_n)d\tau_n. \qquad (1.20)$$

Therefore according to (1.20):

$$\left(\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right\}\hat{}(y), \left\{\prod_{\nu=1}^{n-1} A_1(r_\nu,y_\nu)\right\}\times\right.$$

$$\left.\times A_2(r_n,y_n)\right) =$$

$$= \left(\frac{1}{2\pi}\right)^n \int\limits_{R^n} \overline{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\left[\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right]}\left\{\prod_{\nu=1}^{n-1}\widetilde{A}_1(r_\nu,\tau_\nu)\right\}\times$$

$$\times\widetilde{A}_2(r_n,\tau_n)d\tau_1\dots d\tau_k d\tau_{k+1}\dots d\tau_n =$$

$$= \left(\frac{1}{2\pi}\right)^{n-1}\int\limits_{R^n}\overline{\left[\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right]}\cdot\left\{\prod_{\nu=1}^{n-1}\widetilde{A}_1(r_\nu,\tau_\nu)\right\}\times$$

$$\times\left[\left(\frac{1}{2\pi}\right)\int\limits_{R^1}\overline{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)}\widetilde{A}_2(r_n,\tau_n)d\tau_n\right]d\tau_1\dots d\tau_{n-1} = 0.$$

The assertion 1.1.1 is proved. □

The proved assertion and (1.17) implies that

$$\left(\left\{\gamma(\tau_1,\dots,\tau_k,\tau_{k+1},\dots,\tau_n)\prod_{\lambda=1}^{k} g_\lambda(\tau_\lambda)\right\}\hat{}(y), \varphi_m(l,j,y)\right) = 0$$

and (1.16) implies the assertion of lemma 1.1. □

§2. The Kernel $K(t,a,b)$

This section begins with the construction of the auxiliary function $\Omega(\tau,[-\alpha,\alpha],\beta)$, $\tau \in R^1$, with the help of which the kernel $K(t,a,b)$, $t \in \in R^n$ used in §3 is defined Then the estimation of the kernel norm is obtained in lemma 2.2.

Let us take any $\beta > 0$ and denote by $\omega(\tau,\beta)$ such function, the Fourier transformation of which is

$$\widehat{\omega}(y,\beta) = \frac{1}{\beta^2}\,\xi_{[-\beta/2,\,\beta/2]}(y) * \xi_{[-\beta/2,\,\beta/2]}(y), \tag{2.1}$$

where $\xi_M(y)$ is the characteristic function of the set $M \subseteq R^1$. Then according to (2.1):

$$\widehat{\omega}(y,\beta) = \begin{cases} \dfrac{1}{\beta^2}\,(\beta - |y|), & |y| \le \beta \\ 0, & |y| > \beta \end{cases} \tag{2.2}$$

and

$$\int\limits_{R^1} \widehat{\omega}(y,\beta)\,dy = 1.$$

Let $\alpha > \beta$. Denote by $\Omega(\tau,[-\alpha,\alpha],\beta)$ such function, the Fourier transformation of which is

$$\widehat{\Omega}(y,[-\alpha,\alpha],\beta) = \xi_{[-\alpha,\alpha]}(y) * \widehat{\omega}(y,\beta). \tag{2.3}$$

From (2.1), (2.2) and (2.3) we obtain that

$$\left.\begin{array}{l} 0 \le \widehat{\Omega}(y,[-\alpha,\alpha],\beta) \le 1 \\ \widehat{\Omega}(y,[-\alpha,\alpha],\beta) = 0,\ y \notin (-\alpha-\beta,\alpha+\beta) \\ \widehat{\Omega}(y,[-\alpha,\alpha],\beta) = 1,\ y \in [-\alpha+\beta,\alpha-\beta] \\ \widehat{\Omega}''_{y^2}(y,[-\alpha,\alpha],\beta) \in L^\infty(R^1) \end{array}\right\}. \tag{2.4}$$

For arbitrary vectors $b = (b_1,\dots,b_n)$ and $a = (a_1,\dots,a_n)$ such that $0 < b_k < a_k$, $1 \le k \le n$ let us denote by $K(t,a,b)$, $t \in R^n$ the function, the Fourier transformation of which is

$$\widehat{K}(y,a,b) = \prod_{k=1}^{n} \frac{1}{iy_k}\left[1 - \widehat{\Omega}(y_k,[-a_k,a_k],b_k)\right], \tag{2.5}$$

where $y = (y_1, \dots, y_n)$. From (2.4) it follows that $\widehat{K}(y,a,b) = 0$ for $y \in \in \overline{Q(a-b)}$.

Lemma 2.1. *Let $n \geq 1$, $t = (t_1, \dots, t_n) \in R^n$ $a = (a_1, \dots, a_n)$, $b = (b_1, \dots, b_n)$ are any vectors which coordinates satisfy the following condition: $0 < b_k < a_k$, $1 \leq k \leq n$. Then*

$$K(t,a,b) = \left(\frac{1}{\pi}\right)^n \prod_{k=1}^{n} \int\limits_{\frac{1}{2} b_k t_k}^{\text{sign } t_k \cdot \infty} \frac{\sin\left(2\,\frac{a_k}{b_k}\,\theta\right)\,\sin^2\theta}{\theta^3}\,d\theta. \tag{2.6}$$

Proof. From (2.5) we obtain:

$$K(t,a,b) = \prod_{k=1}^{n} \left\{ \frac{1}{2\pi} \int\limits_{R^1} e^{iy_k t_k} \frac{1}{iy_k} \left[1 - \widehat{\Omega}(y_k, [-a_k, a_k], b_k)\right] dy_k \right\}.$$

Let us choose $k \in \{1, \dots, n\}$. Since

$$\frac{1}{2\pi} \int\limits_{R^1} \frac{e^{iy_k t_k}}{iy_k}\, dy_k = \frac{1}{2}\ \text{sign } t_k$$

and for any $\rho > 0$:

$$\left[\xi_{[-\rho,\rho]}(y_k)\right]\tilde{\ }(t_k) = \frac{1}{2\pi} \int\limits_{R^1} e^{iy_k t_k} \xi_{[-\rho,\rho]}(y_k)\, dy_k = \frac{\sin \rho t_k}{\pi t_k},$$

then from (1.1), (2.1) and (2.3) we have:

$$\frac{1}{2\pi} \int\limits_{R^1} e^{iy_k t_k} \frac{1}{iy_k} \left[1 - \widehat{\Omega}(y_k, [-a_k, a_k], b_k)\right] dy_k =$$

$$= \frac{1}{2}\ \text{sign } t_k - \frac{1}{2}\ \text{sign } t_k * 2\pi \left\{ \left[\xi_{[-a_k,a_k]}(y_k)\right]\tilde{\ }(t_k) \times \right.$$

$$\left. \times \left(\frac{2\pi}{b_k}\right)^2 \cdot \left(\left[\xi_{[-b_k/2,\, b_k/2]}(y_k)\right]\tilde{\ }(t_k)\right)^2 \right\} =$$

$$= \frac{1}{2}\ \text{sign } t_k - \frac{2}{b_k^2 \pi} \int\limits_{R^1} \text{sign}(t_k - \tau)\, \frac{\sin a_k \tau \cdot \sin^2(b_k \tau/2)}{\tau^3}\, d\tau =$$

$$= \frac{1}{2}\ \text{sign}\ t_k - \frac{2}{b_k^2\pi}\ \text{sign}\ t_k \int\limits_{R^1} \frac{\sin a_k\tau \cdot \sin^2(b_k\tau/2)}{\tau^3}\, d\tau +$$

$$+\frac{4}{b_k^2\pi} \int\limits_{t_k}^{\text{sign}\ t_k\cdot\infty} \frac{\sin a_k\tau \cdot \sin^2(b_k\tau/2)}{\tau^3}\, d\tau.$$

After changing the variable in the right-hand side integrals $b_k\tau/2 = \theta$, we obtain:

$$K(t,a,b) = \prod_{k=1}^{n} \left\{ \frac{1}{2}\ \text{sign}\ t_k - \text{sign}\ t_k \cdot \frac{1}{2\pi} \int\limits_{R^1} \frac{\sin(2a_k\theta/b_k)\cdot \sin^2\theta}{\theta^3}\, d\theta + \right.$$

$$\left. +\frac{1}{\pi} \int\limits_{\frac{1}{2}b_k t_k}^{\text{sign}\ t_k\cdot\infty} \frac{\sin(2a_k\theta/b_k)\cdot \sin^2\theta}{\theta^3}\, d\theta. \right\} \tag{2.7}$$

Since according to [14, p. 193, the formula 858.731]:

$$\int\limits_0^{\infty} \frac{\sin(2a_k\theta/b_k)\cdot \sin^2\theta}{\theta^3}\, d\theta = \frac{\pi}{2}, \tag{2.8}$$

then substituting (2.8) in (2.7) we obtain the assertion of this lemma. □

Lemma 2.2. *Let* $n = 1$, $a > 0$, $M \in (1, +\infty)$ *and* $q \in [1, +\infty)$, *then:*

$$\left\| K\left(t, a, \frac{1}{M}\, a\right) \right\|_{L^q(R^1)} = \frac{1}{a^{1/q}}\, C_1(M,q),$$

where

$$C_1(M,q) = \left\{ 4M \int_0^{\infty} \left| \frac{1}{\pi} \int\limits_x^{\infty} \frac{\sin 2M\theta \cdot \sin^2\theta}{\theta^3}\, d\theta \right|^q dx \right\}^{1/q}.$$

Proof. Let us put $n = 1$ and $b = \frac{1}{M}\,a \in R^1$ in (2.6). Then

$$K\left(t, a, \frac{1}{M}\,a\right) = \frac{1}{\pi}\int\limits_{\frac{1}{2M}\,at}^{\operatorname{sign}\, t\cdot\infty} \frac{\sin 2M\theta \cdot \sin^2\theta}{\theta^3}\, d\theta,$$

whence it follows that

$$\left\|K\left(t, a, \frac{1}{M}\,a\right)\right\|_{L^q(R^1)} = \left\{\int\limits_{R^1}\left|\frac{1}{\pi}\int\limits_{\frac{1}{2M}\,at}^{\operatorname{sign}\, t\cdot\infty} \frac{\sin 2M\theta \cdot \sin^2\theta}{\theta^3}\, d\theta\right|^q dt\right\}^{1/q} =$$

$$= \frac{1}{a^{1/q}}\cdot\left\{4M\int\limits_0^\infty\left|\frac{1}{\pi}\int\limits_x^\infty \frac{\sin 2M\theta \cdot \sin^2\theta}{\theta^3}\, d\theta\right|^q dx\right\}^{1/q} = \frac{1}{a^{1/q}}\, C_1(M, q).$$

□

References

[1] Bohr H. *Un théoréme général sur l'intégration d'un polynome trigonométrigue.* Comptes Rendus De L'Academie des sciences, 1935, vol. 200, no. 15, pp. 1276–1277.

[2] Bohr H. *Ein allgemeiner Sats über die Integration eines trigonometrischen Polynomials.* Prace Mathematyzcne Fizyczne, 1935, no. 43, pp. 273–288 (= Collected Mathematical works, 1952, vol. 2, pp. 36)

[3] Favard J. *Application de la formule sommatorie d'Euler á la démonstration de quelques propriétés extremales des integrales des fonctions periodiques on Presqueperiodiques.* Matematisk Tidsskrift. Series B, 1936, pp. 81–94.

[4] Levitan B. M. *On some generalization of S. N. Bernshtein and H. Bohr inequalities.* RAS USSR, 1937, vol. XV, no. 4, pp. 169–172.

[5] Hörmander L. *A new proof and a generalization of an inequality of Bohr.* Mathematica Scandinavica, 1954, vol. 2, pp. 33–45.

[6] Ivanov B. F. *On a generalization of an inequality of Bohr.* The Issues of Analysis, 2013, vol. 2 (20), no. 2, pp. 21–57.

[7] Ivanov B. F. *The sufficient condition of the boundedness and the parameter differentiability of some class linear equations solutions.* Nonlinear dynamical systems. Collection of articles. Is. 1. SPb.: Izdatelstvo SPbSU. SPb., 1997, pp. 125–130.

[8] Ivanov B. F. *Some condition of existence of bounded solutions for one class of nonlinear equations.* Nonlinear dynamical systems – 2007: The theses of the reports of the international congress, Saint-Petersburg, 4–8 June 2007. SPb., Saint-Petersburg State University, 2007, pp. 227.

[9] Gelfand I. M., Shilov G. E. *The generalized functions and the operations over them, iss. 1.* M., PhM, 1959, 470 p.

[10] Vladimirov V. S. *The equations of mathematical physics.* M., Nauka, 1971, 512 p.

[11] Functional analysis. Series: "The reference mathematical library". Ed. by S. G. Krein. M., Nauka, 1972, 554 p.

[12] Kolmogorov A. N., Fomin S. V. *The elements of the functions theory and functional analysis.* M., Nauka, 1968, 496 p.

[13] Makarov B. M., Podkorytov A. N. *The lectures on the real analysis.* SPb., BHV–Petersburg, 2011, 688 p.

[14] Dwight H. B. *Tables of integrals and other mathematical data.* M., Nauka, 1977, 224 p.

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Saint Petersburg State Technological University of Plant Polymers,
Str. Ivan Chernykh, 4, 198095 Saint Petersburg, Russia.
E-mail: ivanov-bf@yandex.ru